Stochastic resonance in bulk semiconductor lasers

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The stochastic time scale of the mode hopping in a bulk semiconductor laser can be varied maintaining the symmetry of the residence times by a proper tuning of the laser substrate temperature and pumping current. While the addition of external noise to the pumping current affects the symmetry of the mode-hopping process, a sinusoidal modulation does not, providing that the modulation amplitude is below a critical value. In this case, we observe stochastic resonance in the modal intensities of the laser. We show the occurrence of the phenomenon in the spectral domain, and we characterize it by a statistical analysis based on the residence times probability distributions. The evidence of *bona fide* resonance is also provided, varying the modulation frequency and analyzing a proper statistical indicator. Changing the temperature of the laser substrate we show that resonance occurs at different modulation periods always equal to the double of the average residence time measured without modulation.

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I. INTRODUCTION

The concept of stochastic resonance has been introduced in the context of ice ages by Benzi *et al.* [1]. The peculiarity of the phenomenon relies on the fact that a finite amount of white, Gaussian noise may induce coherence in a suitable nonlinear system. Such an apparently contradictory idea has proved to be very successful, and much evidence has been provided in different fields (for a review, see, e.g., [2]).

The key ingredient of stochastic resonance, in the classical sense, is the interplay between a bistable system and a small, periodic modulation. In the absence of noise, the driving is unable to induce the switching of the system from a well of the potential to the other. The addition of noise is able to trigger, at a finite level of the noise strength, an almost synchronized jumping of the system between the two wells with the input modulation. Increasing the amount of noise, the quality of the response decreases, thus resulting in a resonance like behavior.

The study of stochastic resonance has been particularly fruitful in nonlinear optics and especially in laser physics, starting from the very first evidence in a real physical system: namely, a ring laser where bistability occurs between two counterpropagating ring cavity modes [3,4]. Recently, the phenomenon has been studied in detail in the polarized emission of vertical cavity lasers [5,6] and in edge-emitting lasers with optical feedback [7].

In this work, we present evidence of stochastic resonance in the modal emission of a bulk, edge-emitting semiconductor laser. For a particular choice of the system parameters—namely, the laser substrate temperature and the pump current—mode hopping between two longitudinal modes is found [8]. The process is stochastic and ruled by the Kramers statistics [9]. We have shown in another work [10] that a suitable path in the parameter space can be chosen in order to vary the average residence times of the active modes (even of several orders of magnitude), but keeping constant their ratio. Such a behavior can be dynamically interpreted as a variation of the stability of the modes, leading to a different sensitiveness to the amount of spontaneous emission noise present in the system. In terms of the phenomenological Langevin description of the particle motion in a bistable potential subject to additive, δ-correlated Gaussian noise (see, e.g., [11]), this could be interpreted as a parametric variation of the potential barrier at a fixed noise level. The resulting process is physically indistinguishable from the one where the potential is kept fixed and the noise strength is varied, and it can lead to the same behaviors.

In our experiment, we show that the addition of a small modulation in the pumping current together with a suitable variation of the parameters for tuning the time scale of the mode hopping is able to produce a resonant response which can be interpreted as a stochastic resonance. Moreover, tuning the modulation frequency and keeping fixed the system parameters, we can show also evidence of a *bona fide* resonance [5,12].

The structure of the paper is the following. In Sec. II we describe the experimental setup. In Sec. III we briefly recall the statistical analysis of mode hopping and we study the effect of an external modulation applied to the pump current. In Sec. IV we describe the occurrence of stochastic resonance in the system, while evidence of *bona fide* resonance is reported in Sec. V. In the last section we draw our conclusions.

II. EXPERIMENTAL SETUP

The experimental setup is very similar to that used in [8]. We monitor the modal intensities of a bulk edge-emitting semiconductor lasers Hitachi Hlp 1400, which is a double-heterostructure $Ga_{1-x}Al_xAs$ laser with a bulk active region and cleaved, uncoated facets. The frequency separation between consecutive longitudinal modes is 125 GHz, and the laser emission is in a single-transverse mode with a wavelength around 840 nm. The lasing threshold is around 80 mA. The laser package temperature is stabilized up to 0.01 °C and the laser current is controlled with a very stable (± 1 μA) power supply. The laser operates in a single-longitudinal mode for the largest region of the parameter space, except close to threshold and in the neighborhood of some current values where a change in the identity of the dominating mode takes place. The origin of these spectral transitions is well known [8,13–15]: when the temperature of the semiconductor medium is increased, by increasing of the laser substrate temperature (T_{sub}) or by increasing of the pumping current J (through the Joule effect), the refractive index of the semiconductor medium increases and produces a slight redshift of the modal wavelengths. At the same time, the peak of the gain spectrum also experiences a redshift. Since the redshift rate of the gain spectrum is larger than the one of the longitudinal modes resonances, the former causes roll-off of the active longitudinal mode m_1 and finally induces the transition to another one at longer wavelength m_2. Close to these transition regions, both modes are stable and the laser displays (noise-induced) mode hopping [8]. Around the transition, for lower currents the modal intensity m_1 is most of the time in the ON state, while the modal intensity m_2 is most of the time in the OFF state, for higher currents the opposite situation is found. Further increase of the bias lead to a stable emission of mode m_2, exiting from the transition region.

III. STATISTICAL PROPERTIES OF THE MODE-HOPPING

In the transition regions, the modal intensities exhibit random switchings from the ON state to the OFF state, featuring a strong degree of anticorrelation (better than −0.95). In [8,10] we have analyzed this modal behavior from a statistical point of view, characterizing the residence times T—i.e., the intervals of emission in a given mode. In the entire parameter region where the modal transition occurs, the residence time probability distribution (RTPD) on each mode is an exponential decaying function, in agreement with the van 't Hoff–Arrhenius law [9,16,17]. This law describes the residence time of a particle in a potential well driven by a Langevin force. Besides, the value of the standard deviation σ_T of the residence time is close to the value of the average residence time ($\overline{T}$), as expected for this kind of statistics. These results indicate that the modal dynamics can be interpreted as the noise-induced movement of a particle in a double potential well, and the decay rate of the RTPD's can be associated to the Kramers time τ_k, which represents the inverse of the average escape rate.

A. Dependence on the laser parameters

The decay rates of the RTPD's for the two modes are in general different. Overlapping of the two distributions, resulting in a symmetrical situation for the statistical properties

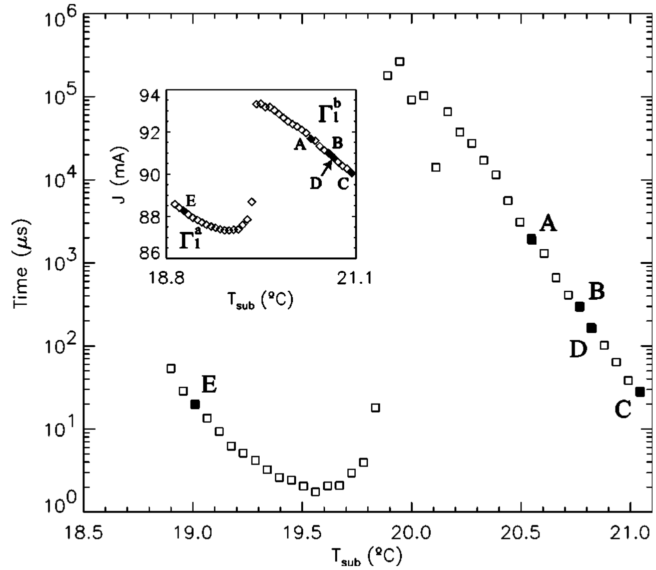

FIG. 1. Average residence time for the modes m_1 and m_2 as a function of the laser substrate temperature T_{sub}. Inset: the path Γ_1 followed in the parameter space (J, T_{sub}) in order to change T_{sub} maintaining the symmetry between the modal emissions (α=1).

of the two modes, is obtained for a specific value of the pumping current and this current value changes as T_{sub} is varied. In order to quantify the degrees of symmetry of the modal emission, in [10] we introduced α as the ratio $\overline{T}_{m_1}/\overline{T}_{m_2}$ between the average residence time of the mode m_1 active below the transition and the mode m_2 active above the transition. We can choose a path in the parameter space (J,T_{sub}), along which α remains constant. Since the two modal intensities are strong anticorrelated, the mode hopping along these

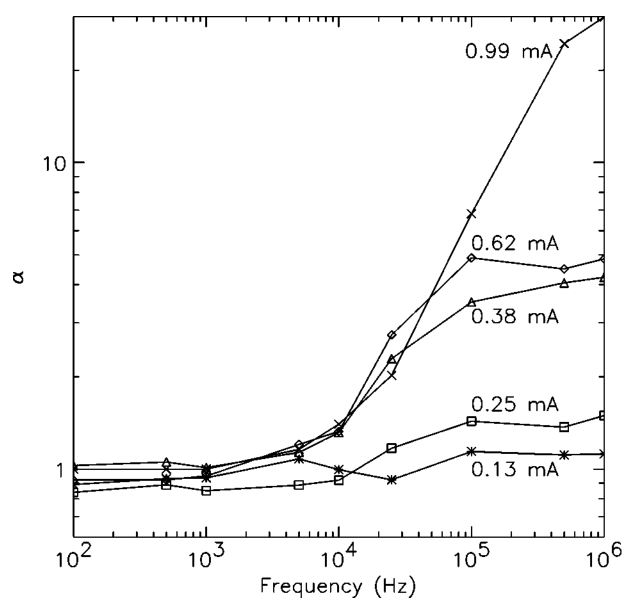

FIG. 2. Symmetry indicator as a function of the modulation frequency in the pumping current for different amplitude rms values. The system is in the parameter conditions corresponding to point B of Fig. 1.

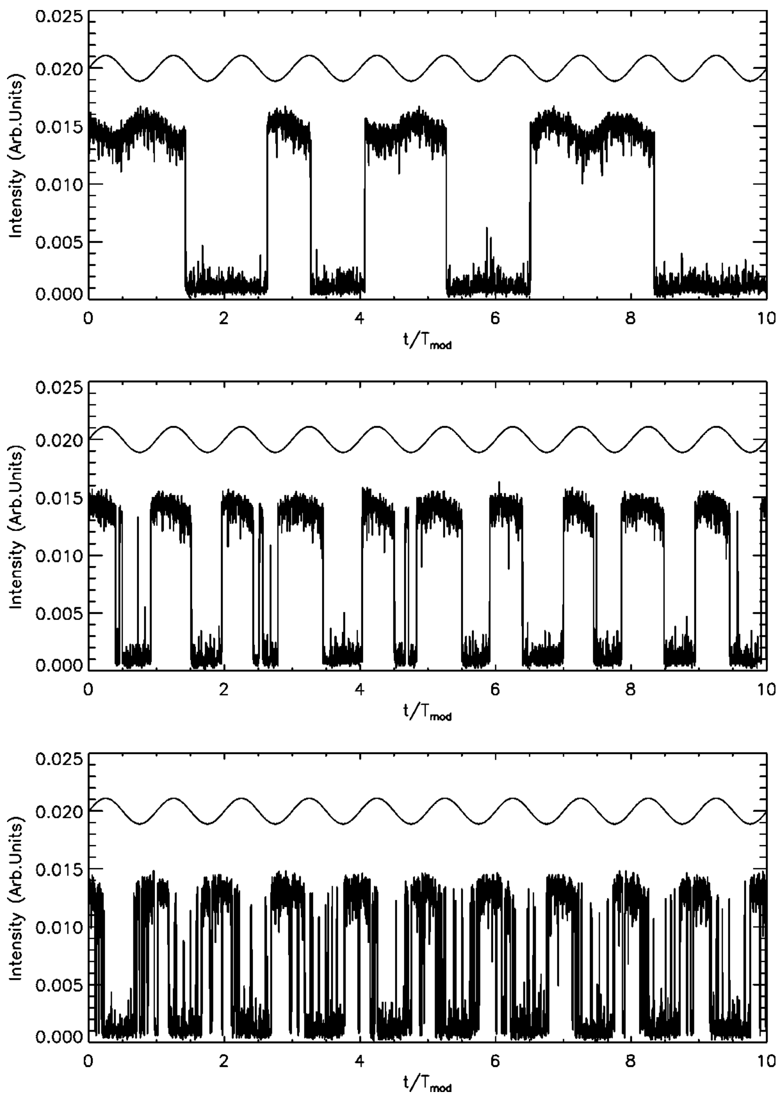

FIG. 3. Modal intensity time series for different parameter conditions (from top to the bottom the parameters correspond to the points marked respectively by A [where $\overline{T}\approx 2\times 10^3$ μs), B (where $\overline{T}\approx 300$ μs), and C (where $\overline{T}\approx 30$ μs) in the branch Γ_1^b plotted in Fig. 1]. The period of the modulation is 500 μs (2 kHz) while its amplitude is 0.25 mA rms.

parameters path can be described by a simple master equation with a single rate. In [10] we have fixed $\alpha=1$, identifying the path Γ_1. We have shown that along Γ_1 it holds an even stronger condition; i.e., the shape of the quasipotential associated to the modal intensities is maintained. This enables the description of the mode-hopping process with a Langevin model with a well-defined potential (derived from the experimentally measured quasipotential) with an additive source of white, Gaussian noise. As we change the parameters moving along Γ_1, the residence times $(\overline{T}_{m_1}=\overline{T}_{m_2}=\overline{T})$ vary accordingly to Fig. 1. The path Γ_1 is described in the

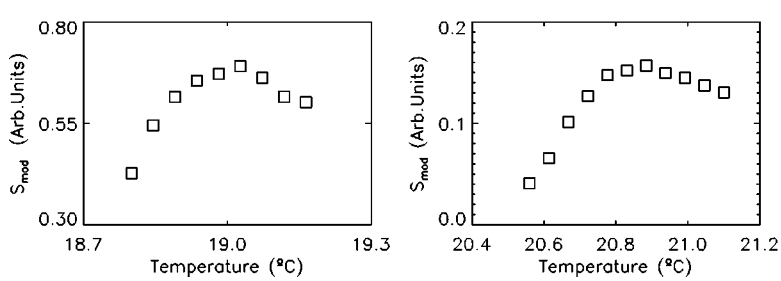

FIG. 4. Height of the peak at the modulation frequency in the modal intensity power spectrum as a function of the position along Γ_1, expressed by the laser substrate temperature. Corresponding pumping current can be obtained from Fig. 1. Left: moving along the branch Γ_1^a, with a modulation in the pumping current of 40 kHz and an amplitude of 0.30 mA rms, Right: moving along the branch Γ_1^b with a modulation in the pumping current of 2 kHz and an amplitude of 0.25 mA rms.

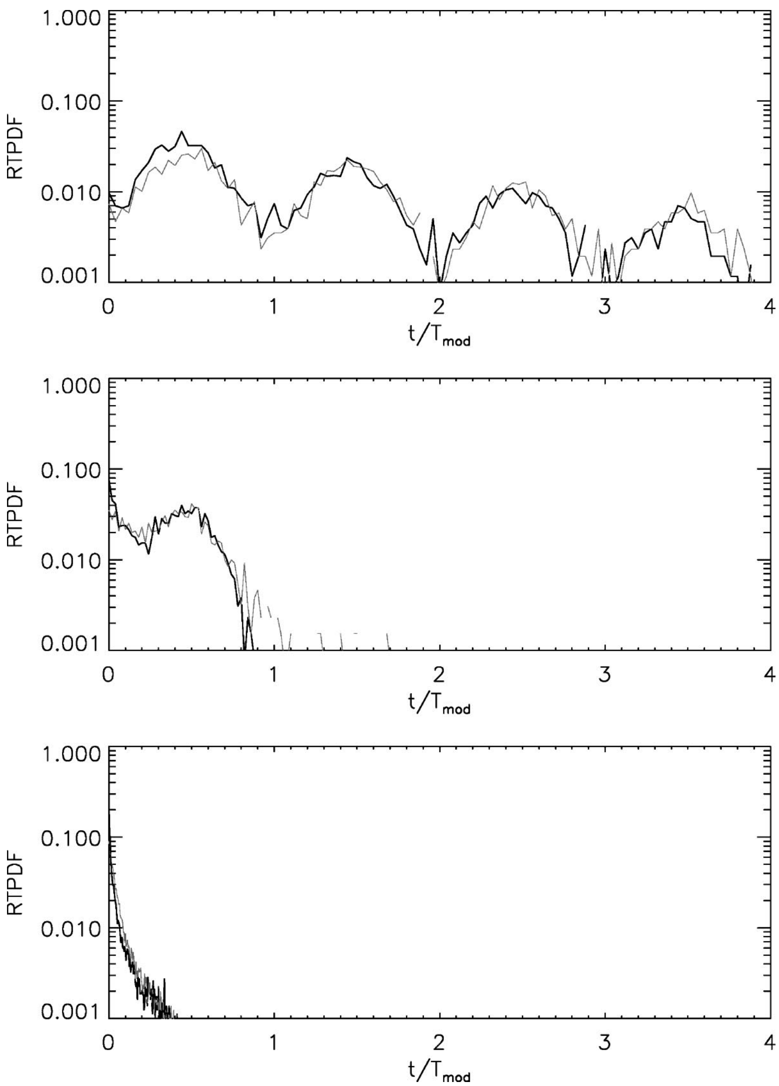

FIG. 5. Residence time probability distribution for the modes active obtained for the same parameter values of Fig. 3 (from top to bottom the parameters correspond, respectively, to the points *A*, *B*, and *C* of Fig. 1). The black trace is relative to the mode m_1; the gray trace is relative to mode m_2.

inset of Fig. 1. We remark the existence of two branches $\Gamma_1^{(a)}$ and $\Gamma_1^{(b)}$ (see Fig. 1) along which $\overline{T}$ varies monotonically with T_{sub}.

The modal intensities probability distributions are well described by the solution of the Langevin equation [11]

$$P(x) = P_0 \exp[-U(x)/D], \tag{1}$$

where P_0 is a normalizing factor, x represents a modal intensity, $U(x)$ is the bistable potential, and the system is driven by a white, Gaussian noise source $\xi(t)$ such that $\langle \xi(t)\xi(t')\rangle = 2D \times \delta(t-t')$, where $\langle \cdots \rangle$ stands for time integration.

We point out that, as shown in [10], the variations of $\overline{T}$ along Γ_1 (Fig. 1) are due to a modification of the quasipotential $U(x)$ by a scaling factor, while the phenomenological noise D remains constant.

The possibility of varying the hopping rate, maintaining the symmetry of the potential, allows for tuning the stochastic time scale of the system to the (half) period of an external modulation, which is the standard condition for observing the stochastic resonance phenomenon.

B. Effect of externally added noise

Another method for changing the hopping time scale is by adding electrical noise on the pumping current. This method relies on the fact that, as shown before, J influences deeply the residence times on each mode. In a similar system [5], indeed, this technique was successfully implemented. Anyway, considering the field variable, pumping current enters into the laser field equations in a multiplicative way. Therefore, the equivalence between adding noise to the pumping current and introducing additively noise into the system is far from being trivial and it must be verified. In [10] we show that the symmetry of the modal emission is strongly affected by increasing the noise level added to J. This result cannot be interpreted in the frame of additive noise; using the Langevin picture of the (symmetric) double potential well, an increase of the noise level D should alter equally the residence times of both wells. Then, we conclude that the pumping current cannot be used straightforwardly in our system as the channel for injecting additive noise.

C. Response to an externally added modulation

In order to explore the possibility of observing stochastic resonance we analyze how mode hopping is affected by ad-

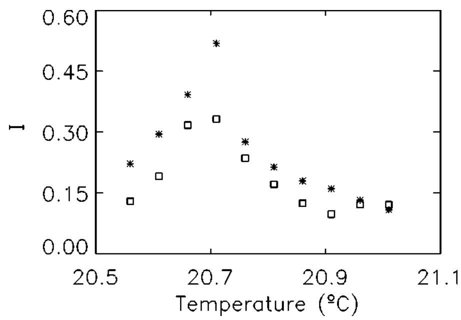

FIG. 6. Indicator I as a function of the position along Γ_1, expressed by the laser substrate temperature. Corresponding current value can be induced from Fig. 1. Squares are obtained for the modal intensity m_1, while stars for the modal intensity m_2. The modulation characteristics are as in Fig. 3.

dition of an external periodic modulation. In our system we have shown that the residence times on each mode are deeply affected by the pumping current, which, therefore, represents the most natural channel for introducing a modulation.

Stochastic resonance phenomenon is induced by a subthreshold modulation amplitude; i.e., the external forcing is unable to induce synchronized jumps between the two wells in the absence of noise. In our system, since we cannot change experimentally the internal noise level, this condition on the modulation amplitudes A_{mod} has been verified using an alternative method. We have checked that, for modulation frequencies (ν_{mod}) much lower than the Kramers rates, the amplitudes A_{mod} used are small enough that there is no significative interaction between the modulation and the modal behavior. This means that the hopping statistics of the system modulated at $\nu_{mod} \ll (\tau_k)^{-1}$ are similar to the ones of the system unforced; i.e., the RTPD's remain exponentially decaying functions with the same Kramers rates.

Another requirement is that the external forcing should not modify adiabatically the symmetry of the quasipotential. This has been verified checking how the symmetry indicator α is affected by A_{mod}. In Fig. 2, we plot α for different frequencies of the modulation and we observe that α remains unaffected in the range of frequencies explored for modulation amplitude smaller than 0.25 mA rms. Above this value, α becomes dependent on ν_{mod}. The quantitative result of Fig. 2 is particular of the transition explored and on the parameter values chosen (point B of Fig. 1). For the transition studied and for the entire path Γ_1, this condition on the modulation amplitude is more restrictive than the condition for subthreshold excitation. In practice, when modulating the system, if α remains unchanged for the frequencies of the forcing used, both conditions are verified.

IV. STOCHASTIC RESONANCE

We investigate the response of the system by applying a sinusoidal modulation with a subthreshold amplitude at a period T_{mod}. Then, we vary the average mode-hopping duration $\bar{T}$ by changing the parameters according to the path Γ_1. In the following, we will refer to the parameters used in the experiment by indicating the value of T_{sub}; the corresponding value for J (and of $\bar{T}$) can be induced from Fig. 1. As a rule, a resonant behavior is expected when $T_{mod} \approx 2\bar{T}$. Choosing a T_{mod} compatible with the hopping time scales described in Fig. 1, the temporal behavior of a modal intensity is shown for the branch $\Gamma_1^{(b)}$ in Fig. 3. For a different T_{mod}, a similar response is found along the second branch $\Gamma_1^{(a)}$. If T_{mod} is chosen such that it is compatible with the time scales of both branches, a double resonance can be found by increasing T_{sub} along Γ_1.

Analyzing the modal time series (Fig. 3, from bottom to top), we observe that, if $2\bar{T} < T_{mod}$, the system exhibits random-mode hops not synchronized with the forcing, and if $2\bar{T} \approx T_{mod}$, the mode hopping occurs synchronously with the modulation and we have a maximum of coherence in the modal signal. Finally, for $2\bar{T} > T_{mod}$, mode hopping does not occur anymore at every half-period of the forcing and synchronization may be present but only for time windows lasting few modulation periods.

A quantitative description of the resonance can be performed by plotting the magnitude of the modal signals component at the frequency of the modulation. The result is shown in Fig. 4, where we plot the response along the branch $\Gamma_1^{(a)}$ (left) and $\Gamma_1^{(b)}$ (right).

A clear maximum can be noticed in both cases when the resonance condition $T_{mod} \approx 2\bar{T}$ is fulfilled. This is a clear evidence of the stochastic resonance phenomenon, obtained by proper tuning of the stochastic time scale of the system to the external modulation period. It is worth noting that, at variance with conventional stochastic resonance, where the inter-

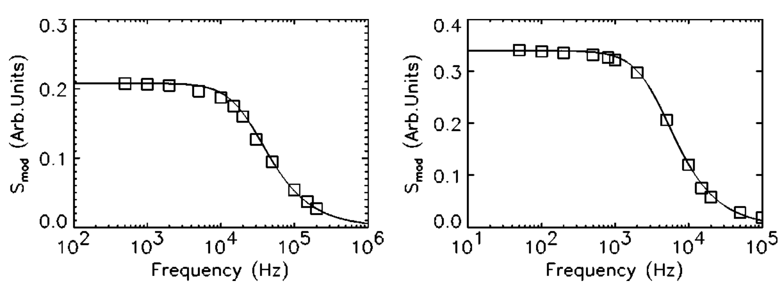

FIG. 7. Height of the peak at the modulation frequency in the modal intensity power spectrum as a function of the modulation frequency. The solid lines are a fit with single-pole response functions. In the left panel the system parameters correspond to the point E (for which $\bar{T} \approx 20$ μs) in the branch Γ_1^a plotted in Fig. 1. In the right panel the system parameters correspond to the point D (for which $\bar{T} \approx 160$ μs) in the branch Γ_1^b plotted in Fig. 1.

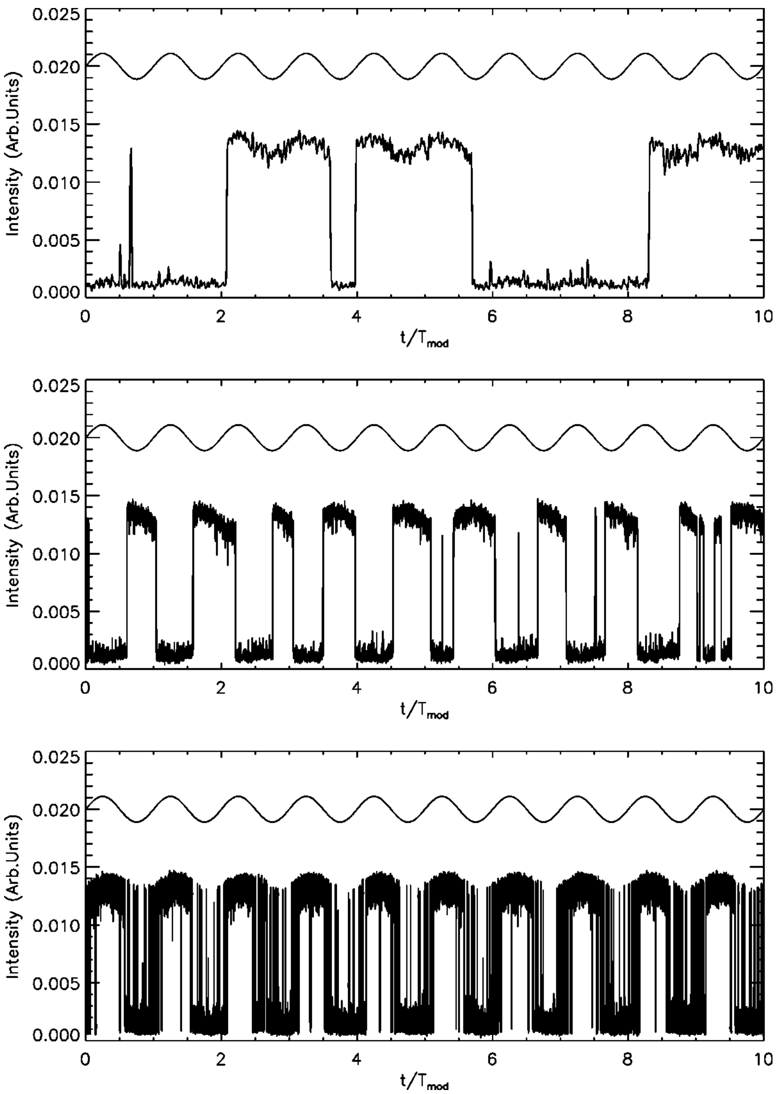

FIG. 8. Left: modal intensity time series for different modulation frequencies at the temperature of 20.5 °C, corresponding to point D on the parameter path plotted in Fig. 1. From top to bottom: 20 kHz, 2 kHz, 0.1 kHz. The amplitude of the modulation is 0.25 mA rms. Since in the point D $\overline{T}\approx 160\ \mu s$, we have that $(2\overline{T})^{-1}\approx 3.1$ kHz.

nal time scale is tuned by addition of external noise, we are able to obtain this phenomenon relying on the dependence of the internal time scale of the system with its parameters.

An useful analysis of the observed behavior can be performed by considering the activation residence times T of a mode, defined as the time interval between a switching-on event and a switching-off event of this mode, and perform the corresponding statistics [2]. In Fig. 5, we plot the residence time probability distributions corresponding to the time series plotted in Fig. 3, evaluated for both modes. For $2\overline{T}<T_{mod}$ the RTPD's are exponentially decaying functions, as a consequence of the random hopping. For $2\overline{T}>T_{mod}$, the RTPD's exhibit several peaks corresponding to odd multiple of $T_{mod}/2$, indicating that the hopping is not synchronous with the modulation but it may occur over few modulation periods. Finally, for $2\overline{T}\approx T_{mod}$ the RTPD's present a single peak, showing that most of the residence times are comparable with half of the modulation period and a statistical synchronization could be present.

In order to quantify the resonance in this context, an indicator has been introduced measuring the excess of jumps synchronized to the forcing with respect to the jumps distribution without modulation [5]. This indicator is defined as

$$I=\int_{T_{mod}/2-\beta T_{mod}}^{T_{mod}/2+\beta T_{mod}}\tilde{P}(T)dT, \qquad (2)$$

where $\tilde{P}(T)$ is the RTPD with substraction of the background [5] and $0<\beta<1/4$ define the width of the integration region. In Fig. 6 we plot, for both modes, I as a function of the position of the system along the parameter path Γ_1^b. The presence of a maximum for I when $2\overline{T}\approx T_{mod}$ is a further strong indication of the occurrence of stochastic resonance.

V. *Bona fide* RESONANCE

A standard procedure for characterizing a system is the analysis of its frequency response to a periodical forcing. This is particularly relevant for semiconductor lasers in view of their telecom application. In our case, chosen a point in the path Γ_1 with an average residence time $\overline{T}$ on both mode, we vary the period of the modulation applied. The frequency response of the modal intensity is plotted in Fig. 7 for two working points (see Fig. 1): E in the branch $\Gamma_1^{(a)}$, for which

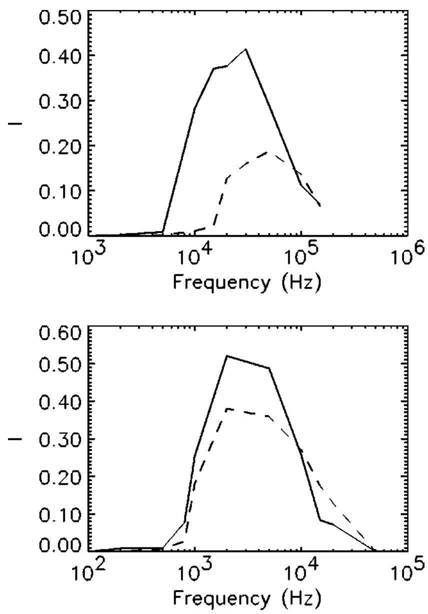

FIG. 9. Indicator I as a function of the modulation frequencies. In the left panel the system parameters correspond to the point E [for which $(2\overline{T})^{-1} \approx 25$ kHz] on the path plotted in Fig. 1. In the right panel the system parameters correspond to the point D [for which $(2\overline{T})^{-1} \approx 3.1$ kHz] on the path plotted in Fig. 1. Dashed lines are relative to the modal intensity m_1, while solid lines are relative to the modal intensity m_2.

$\overline{T} \approx 20$ μs $[(2\overline{T})^{-1} \approx 25$ kHz$]$ and D in the branch $\Gamma_1^{(b)}$, for which $\overline{T} \approx 160$ μs $[(2\overline{T})^{-1} \approx 3.1$ kHz$]$. The curves show a single-pole behavior, with a cutoff frequency at about 26 kHz for the path $\Gamma_1^{(a)}$ and at about 3.8 kHz for the path $\Gamma_1^{(b)}$. No resonance can be evidenced by this kind of analysis.

On the other hand, plotting the modal intensity for different values of T_{mod} evidences that for $T_{mod} \approx 2\overline{T}$ the modal intensity exhibits a maximum of coherence (see Fig. 8). For such a modulation frequency mode hopping occurs synchronously with the forcing.

The enhancement of the response of the system with the forcing for a given frequency together with the single-pole frequency response suggests that this behavior can be interpreted as a *bona fide* resonance [12]. Such a phenomenon relies on the degradation of the signal response at low frequencies, due to random hopping within half of the modulation period (see bottom part of Fig. 8). While the coherence of the signal is strongly affected by those events, the Fourier response is not significatively influenced, resulting in a flat response for low frequencies. As a consequence, in order to underline and quantify the phenomenon it is necessary to evaluate the RTPD's and to analyze the value of the indicator I [Eq. (2)] as a function of modulation frequency. Such a method has been proven to be effective in the evidence of *bona fide* resonance in vertical cavity lasers [5]. We report in Fig. 9 the result for the two points analyzed. A clear maximum of I is found in both cases, thus confirming the existence of *bona fide* resonance in our system.

We have verified that *bona fide* resonance can be observed for all the points of the path Γ_1, thus varying the resonating value of T_{mod} on several orders of magnitude, as shown in Fig. 10. We remark that the points obtained are close to the line for which $T_{mod} \approx 2\overline{T}$, as expected.

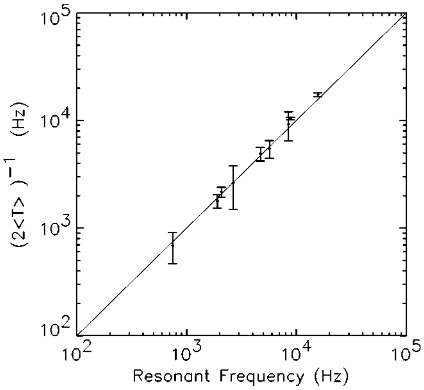

FIG. 10. The inverse of the double of the average residence times (measured removing the modulation) plotted as a function of the corresponding values of the modulation frequency that maximize the indicator I (calculated on the modal intensity m_1; a similar curve holds for the other mode).

In [10] we have pointed out that the average value of the modal residence time is related to the stability of the longitudinal modes and it depends on the laser parameters T_{sub} and J. In the frame of the Langevin description, the residence time has been associated to the depth of the potential wells rather than to the noise level which is not affected significantly by the parameters. In other words, we vary the stochastic time scale of the hopping by scaling the potential function instead of simply change the noise level. This makes the stochastic resonance observed in our system rather peculiar with respect to the conventional ones.

VI. CONCLUSIONS

We have analyzed the stochastic dynamics of mode hopping in bulk semiconductor lasers. This experimental investigation has revealed that stochastic resonance can be observed by modulating the pumping current and properly tuning the parameters of the system. In such a way, the average residence times of the modal intensities change along a path in the parameter space (T_{sub}, J), matching the modulation half-period and leading to the resonance. While the phenomenon is noise induced, the stochastic time scale of the hopping is modified by a scaling of the potential function instead by a variation of the noise level. The phenomenon has been observed in different points of the path and evidenced both in the spectral domain and with a statistical characterization based on the RTPD's. Evidence of *bona fide* resonance has also be provided by modulating the system at different frequencies while its parameters were kept constant.

ACKNOWLEDGMENTS

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[1] R. Benzi, A. Sutera, and A. Vulpiani, J. Phys. A **14**, 453 (1981); C. Nicolis and G. Nicolis, Tellus **33**, 225 (1981).

[2] L. Gammaitoni, P. Hänggi, P. Jung, and F. Marchesoni, Rev. Mod. Phys. **70**, 223 (1998).

[3] B. McNamara, K. Wiesenfeld, and R. Roy, Phys. Rev. Lett. **60**, 2626 (1988).

[4] R. Roy, R. Short, J. Durnin, and L. Mandel, Phys. Rev. Lett. **45**, 1486 (1980).

[5] G. Giacomelli, F. Marin, and I. Rabbiosi, Phys. Rev. Lett. **82**, 675 (1999); S. Barbay, G. Giacomelli, and F. Marin, Phys. Rev. E **61**, 157 (2000).

[6] M. B. Willemsen, M. P. van Exter, and J. P. Woerdman, Phys. Rev. Lett. **84**, 4337 (2000).

[7] F. Marino, M. Giudici, S. Barland, and S. Balle, Phys. Rev. Lett. **88**, 040601 (2002).

[8] L. Furfaro, F. Pedaci, X. Hachair, M. Giudici, S. Balle, and J. Tredicce, IEEE J. Quantum Electron. **40**, 1365 (2004).

[9] H. Kramers, Physica (Utrecht) **7**, 284 (1940).

[10] F. Pedaci, M. Giudici, G. Giacomelli, and J. R. Tredicce Eur. Phys. J. D (to be published).

[11] N. G. Van Kampen, *Stochastic Processes in Physics and Chemistry*, North-Holland Personal Library (Elsevier Science, Amsterdam, 1997).

[12] L. Gammaitoni, F. Marchesoni, and S. Santucci, Phys. Rev. Lett. **74**, 1052 (1995).

[13] R. Linke, B. Kasper, C. Burrus, I. Kaminow, J. Ko, and T. Pei Lee, IEEE J. Lightwave Technnol. **3**, 706 (1985).

[14] M. Ohtsu, Y. Teramachi, Y. Otsuka, and A. Osaki, IEEE J. Quantum Electron. **22**, 535 (1986).

[15] M. Ohtsu and Y. Teramachi, IEEE J. Quantum Electron. **25**, 31 (1989).

[16] J. van 't Hoff, *Etude de Dynamiques Chimiques*, (Muller, Amsterdam, 1884).

[17] S. Arrhenious, Z. Phys. Chem. (Leipzig) **4**, 226 (1889).